HAUSDORFF SCHOOL
FOR ADVANCED STUDIES
IN MATHEMATICS

UNIVERSITÄT BONN

**Hausdorff School**

# "Diffusive Systems: Pattern Formation, Bifurcations, and Biological Applications"

**12 to 15 April 2021**

organized by
**Esther S. Daus, Annalisa Iuorio, Cinzia Soresina**

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## Abstracts

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**José A. Carrillo** (University of Oxford)

### Aggregation-Diffusion equations and systems in mathematical biology: stationary states, phase transitions and qualitative behavior

**Abstract:** We discuss microscopic and continuum cell-cell adhesion models and their derivation based on the underlying microscopic assumptions. We will derive these macroscopic limits via mean-field assumptions. We propose an improvement on these models leading to sharp fronts and intermingling invasion fronts between different cell type populations. The model is based on basic principles of localized repulsion and nonlocal attraction due to adhesion forces at the microscopic level. We also review the applications of these models in cell sorting in developmental biology. We will analyse the mathematical properties of the resulting aggregation-diffusion and reaction-diffusion systems based on variational tools. The concept of H-stability of the interaction potential plays an important role in the appearance of phase transitions in these models.

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**Anna Marciniak-Czochra** (Heidelberg University)

### Pattern formation in reaction-diffusion-ODE systems: Stability analysis and emergence of singularities

**Abstract:** Classical mathematical models of biological or chemical pattern formation have been developed using partial differential equations of reaction-diffusion type. The lectures will be devoted to mathematical analysis of a special class of pattern formation models coupling reaction-diffusion equations with ordinary differential equations. Such systems of equations arise from applications in biosciences and describe interactions between diffusive and immobile agents, for example diffusing growth factors and spatially localised cellular processes. We will focus on two-component reaction-diffusion and reaction-diffusion-ODE systems which serve as basic models to understand pattern formation mechanisms. Pattern formation ability of such models will be discussed in the context of the classical theory of Turing-type pattern formation (based on diffusion-driven instability of spatially homogenous steady states), and more recent results based on existence of multiple steady states and hysteresis. In particular, we will show that in such models all close-to-equilibrium (Turing) patterns are unstable and explain mechanisms of emergence of spatially heterogenous structures with jump

discontinuities (far-from-equilibrium patterns). Finally, we will show that in some cases diffusion may lead to unbounded growth of solutions and mass concentration. The applied methods of model analysis will involve two-point boundary value problems, semigroup theory, spectral analysis and singular perturbation methods. The established mathematical theory and related models will be presented in context of symmetry breaking and pattern formation in developmental biology.

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**Mariya Ptashnyk** (Heriot-Watt University)

**Multiscale modelling and analysis of biological systems**
**Lecture (Talk 1): Locally-periodic and stochastic homogenization**
**Lecture (Talk 2): Multiscale modelling and analysis of tissue biomechanics**

**Abstract:** In the first lecture, we shall consider the main methods and techniques of locally-periodic and stochastic homogenization and apply them to connect microscopic (cell level) and macroscopic (tissue level) descriptions of different biological processes, e.g. chemotaxis, intercellular signalling and transport processes. In the second lecture, we shall consider microscopic modelling of tissue biomechanics and use multiscale analysis techniques to derive macroscopic models from the microscopic description of the underlying processes. The macroscopic models can then be simulated using standard numerical methods.

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**Vivi Rottschäfer** (Universiteit Leiden)

**Pattern formation in reaction-diffusion systems**

**Abstract:** In this lecture series, we consider and analyse patterns that arise when basic, steady state solutions become unstable. Moreover, we assume that this instability appears because of the presence of the diffusion terms. The patterns that occur through this mechanism are small amplitude solutions that oscillate around the steady state.
First, I will give an introduction to the general analysis of this phenomenon in reaction-diffusion systems. There I introduce the so-called Turing bifurcation. This Turing bifurcation occurs in many systems and equations, not only in those of reaction-diffusion type. Its presence and the implications of this has been widely studied.
Next, I will apply this method to systems where mass is conserved. Systems are called conserved if neither mass is created nor destroyed within the system. These type of systems are abundant in nature. An example is a system of grazers like cows. In this system there exist two types of cows: grazing cows and moving cows. The system is conserved because the number of cows does not change. The herbivores prefer to graze in places with low vegetation, and therefore, they aggregate towards these places.
For a general mass conserved system, I will present the analysis leading to a Turing bifurcation. This analysis differs from that of the normal, non-conserved case. Also, I will show patterns that arise because of the presence of the Turing bifurcation. Apart from this, I will also briefly consider other patterns that arise far from equilibrium.

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